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The Optimal Control Model Use for Remove Mold from Homes

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Abstract

In this paper we consider a mathematical model of mold $x(t)$ and Fungicide $u(t)$. The aim of this model is permanently remove mold from houses. Molds can cause mild to severe health problems in sensitive individuals when a sufficient number of airborne spores are inhaled. We used the optimal spray strategies to remove amount of mold. We work optimal control framework by applying the Pontryagin's Maximum Principle. A characterization of the optimal control via adjoint variables was established. We obtained an optimality system that we sought to solve numerically by used MATLAB program.

المستخلص

في هذا البحث نعتبر نموذجا رياضيا للعفن $x(t)$ ومبيد الفطريات $u(t)$ الهدف من هذا النموذج هو إزالة العفن بشكل دائم من المنازل .يمكن أن تسبب العفن مشاكل صحية خفيفة إلى شديدة لدى الأفراد الحساسين عند استنشاق عدد كاف من الجراثيم المحمولة جوا . استخدمنا استراتيجيات الرش المثالية لإزالة كمية العفن .نحن نعمل بإطار تحكم مثالي من خلال تطبيق مبدأ بونتريا جن الأقصى .تم وضع توصيف للتحكم الأمثل عبر المتغيرات المجاورة .لقد حصلنا على نظام أمثل سعينا إلى حله عدديا باستخدام برنامج MATLAB

Keywords: ***Optimal Control Model, Pontryagin's Maximum Principle, algorithm, MATLAB.***

1-Introduction

1.1 General Introduction

Molds are fungi, usually microscopic in size, that occur in nature in large quantities. They reproduce by releasing spores into the air that settle on surfaces and, under the right conditions, grow. Growths of mold can often be seen in the form of a discoloration, ranging from white to orange and green to brown and black. Mold can sometimes be detected by its musty odor. Mildew is a common mold. When mold spores settle on organic or contaminated surfaces and when other conditions of temperature, humidity, shade or darkness, and oxygen supply are conducive, they germinate and develop new colonies of mold. Even surfaces from which mold has previously been removed can have mold growing again if the conditions are right.

Molds thrive on organic materials like natural fibers (such as cotton and wool), paper, leather, wood, or surfaces coated with the slightest amount of organic matter such as food, grease, and soil. Molds that continue to grow can eventually eat away the organic medium that is their source of food.

Wooden structural materials and textiles can deteriorate when mold is allowed to thrive on them.

Molds grow best in warm temperatures, 77 to 86 degrees Fahrenheit, though some growth may occur anywhere between 32 and 95 degrees.

Molds require moisture. Moisture can come from water leaks, flooding, capillary movement (wicking from one area to another), high relative humidity, and condensation. The moisture may be in the host material, on its surface, or in the form of humidity in the air. Relative humidity levels above 70 percent appear to be optimal for fungal or mold spore growth. A lower relative humidity level reduces the rate of mold growth as the mold goes dormant but does not stop growth and development entirely. In fact, at low relative humidity levels, there is increased spore release into the air.

We are all exposed to many kinds of mold both inside and outside the house. However, some people seem to be more sensitive to mold and have allergies to some types of mold. These people may suffer from cold-like Symptoms. When people are experiencing these symptoms, it is difficult to know if they are the result of exposure to molds or have other causes. When breathed,

some mold spores are small enough to go deeply into the lungs and cause serious illness. It is not healthy to live in a home with high levels of mold.

Many times, mold can be detected by a musty odor. Although mold spores are too small to be seen, colonies of mold growth are sometimes visible on damp walls and musty-smelling textiles. Mildew is one type of mold that can often be seen. In most cases, it is not practical to test for mold growth in a house. There are no standards for "acceptable" levels of mold in a dwelling, and when testing is done, it is usually to compare levels of mold spores inside the house with levels outside the house. It is generally better to look for mold in those places where conditions promote mold growth.

Generally, mold may be found any place where moisture or relative humidity levels are high. Wet or damp **basements** may have mold growing on the walls, floors, carpeting, or on materials stored in the basement.

Moisture from the earth can migrate through concrete walls causing them to remain damp. Water standing in sump holes, condensate from an air conditioner or dehumidifier, leaky pipes, or water seeping into the basement are all sources of moisture that can support mold growth. Basement carpeting often has mold growing on or under it if the carpeting is installed on a concrete floor that remains cool and damp. Materials stored in a damp basement may have mold growing on them. In particular, firewood stored in the basement puts moisture into the air and is an excellent medium for mold growth. The mold spores can then spread throughout the house. **Crawlspaces** built over uncovered earth can have mold problems when the moisture in the ground causes dampness in the space. Crawlspaces that are sloped incorrectly and have water pooling in them are particularly likely to have problems.

Mold can often be found growing in the **bathroom**. If an exhaust fan is not used during bathing, large amounts of moisture can remain in the shower or tub area. Soap scum on bath and shower walls, even on ceramic tile or fiberglass, is a nutrient source for mold growth. In the **laundry room**, unvented clothes drying produces high levels of relative humidity that support mold growth. Damp towels and clothes in laundry hampers, washers, or dryers can develop mildew growth. Using a **humidifier** sometimes raises the relative humidity high enough that mold will grow. Particularly in the winter, high relative humidity in areas where there is little air movement

results in condensation on cold walls and subsequent mold growth. Dark patches of mold can sometimes be seen inside the upper corner of a closet on an outside wall or behind furniture placed against outside walls. Window condensation can result in mold growth where the moisture runs onto the sill or wood trim. Mold growth can be found on **kitchen** walls if household cooking involves large amounts of boiling water and no exhaust fan is used. The cooking spatters and grease film on walls are the source of nutrients for the mold, combined with the high humidity levels in those areas. Floor-level pans that collect the condensate from automatic defrosting refrigerators often have mold growing in them. **New construction materials**, such as new wooden wall studs and floor joists, drywall compound, and masonry materials emit moisture into the home while the construction components dry. **Unvented combustion heaters**, such as kerosene heaters, emit large amounts of humidity into the air with the exhaust gases [1, 2, 3,4].

Fungicide is a chemical substance that destroys or inhibits the growth of fungi.

1.2 The mathematical model

In our basic optimal control problem for ordinary differential equations, we use $u(t)$ for the control and $x(t)$ for the state. The state variable satisfies a differential equation which depends on the control variable:

$$x'(t) = g(t, x(t), u(t))$$

As the control function is changed, the solution to the differential equation will change. Thus, we can view the control-to-state relationship as a map $u(t) \rightarrow x = x(u)$ (of course, x is really a function of the independent variable t, we write x (u) simply to remind us of the dependence on u). Our basic optimal control problem consists of finding a piecewise continuous control $u(t)$ and the associated state variable $x(t)$ to maximize the given objective functional, i.e

$$\max_u \int_{t_0}^{t_1} f\big(t, x(t), u(t)\big)dt$$

$$\text{Subject to } x'(t) = g(t, x(t), u(t))$$

$$x(t_0) = x_0 \, and \, x(t_1) free.$$

Such a maximizing control is called an optimal control. By $x(t_1)$ free, it ismeant that the value of $x(t_1)$ is unrestricted. For our purposes, f and g will always be continuously differentiable

functions in all three arguments. Thus, as the control(s) will always be piecewise continuous, the associated states will always be piecewise differentiable [5].

1.3 Pontryagin's maximum principle

$$H(t,x,u,\lambda) = f(t,x,u) + \lambda g(t,x,u)$$

= integrand + adjoint * RHS of DE*:*

We are maximizing *H* with respect to *u* at u^*, [6, 7] and the above conditions can be written in terms of the Hamiltonian:

$$\frac{\partial H}{\partial u} = 0 \; at \; u^* \Rrightarrow f_u + \lambda g_u \qquad (\textit{Optimality condition}),$$

$$\lambda' = -\frac{\partial H}{\partial x} \Rrightarrow \lambda' = -(f_x + \lambda g_x) \qquad (\textit{adjoint equation}),$$

$$\lambda(t_1) = 0 \qquad (\textit{transversality condition}).$$

We are given the dynamics of the state equation:

$$x' = g(t,x,u) = \frac{\partial H}{\partial \lambda}, x(t_0) = x_0.$$

Theorem

Consider

$$J(u) = \int_{t_0}^{t_1} f\big(t, x(t), u(t)\big)dt$$

$$\text{Subject to } x'(t) = g\big(t, x(t), u(t)\big), \quad x(t_0) = x_0$$

Suppose that $f(t,x,u)$ *and* $g(t,x,u)$ are both continuously differentiable functions in their three arguments and concave in *x* and *u*. suppose u^* is a control, with associated state x^*, and λ a piecewise differentiable function, such that u^*, x^*, [5, 6] and λ together satisfy on $t_0 \le t \le t_1$:

$$f_u + \lambda g_u = 0,$$
$$\lambda' = -(f_x + \lambda g_x),$$
$$\lambda(t_1) = o,$$
$$\lambda(t) \ge 0.$$

Then for all controls u, we have

$$J(u^*) \ge J(u).$$

Proof

Let u be any control, and x its associated state. Note, as $f(t,x,u)$ is concave in both the x and u variable, we have by the tangent line property

$$f(t,x^*,u^*) - f(t,x,u) \geq (x^* - x)f_x(t,x^*,u^*) + (u^* - u)f_u(t,x^*,u^*)$$

This gives

$$J(u^*) - J(u) = \int_{t_0}^{t_1} f(t,x^*,u^*) - f(t,x,u)$$

$$\geq \int_{t_0}^{t_1} (x^*(t) - x(t))f_x(t,x^*,u^*) + (u^*(t) - u(t))f_u(t,x^*,u^*)dt \ldots\ldots\ldots (1)$$

Substituting

$f_x(t,x^*,u^*) = -\lambda'(t) - \lambda(t)g_x(t,x^*,u^*)$ and

$$f_u(t,x^*,u^*) = -\lambda(t)g_u(t,x^*,u^*)$$

as given by the hypothesis, the last term in (1) becomes

$$\int_{t_0}^{t_1} \big(x^*(t) - x(t)\big)\big(-\lambda'(t) - \lambda(t)g_x(t,x^*,u^*)\big)dt + \big(u^*(t) - u(t)\big)\big(-\lambda(t)g_u(t,x^*,u^*)\big)dt.$$

Using integration by parts, and recalling , $\lambda(t_1) = 0$ and $x(t_0) = x^*(t_0)$, we see

$$\int_{t_0}^{t_1} -\lambda'(t)\big(x^*(t) - x(t)\big)dt = \int_{t_0}^{t_1} \lambda(t)(x^*(t) - x(t))'\,dt$$

$$= \int_{t_0}^{t_1} \lambda(t)(g\big(t,x^*(t),u^*(t)\big) - g\big(t,x(t),u(t)\big)dt.$$

Making this substitution,

$$J(u^*) - J(u) \geq \int_{t_0}^{t_1} \lambda(t)[g(t,x^*,u^*) - g(t,x,u) - (x^* - x)g_x(t,x^*,u^*)$$

$$+ (u^* - u)g_u(t,x^*,u^*)]dt$$

Taking into account $\lambda(t) \geq 0$ and that g is concave in both x and u, this gives the desired result

$$J(u^*) - J(u) \geq 0.$$

2-Results and discussion

2.1 The optimal control problem

Let x (t) be a population concentration at time t, and suppose we wish to reduce the population over a fixed time period. We will assume x has a growth rate r and carrying capacity M. The application of a substance is known to decrease the rate of change of x, by decreasing the

rate in proportion to the amount of *u* and *x*. Let *u* (*t*) be the amount of this substance added at time *t*. For example, the population could be an infestation of an insect, or a harmful microbe in the body. Here we view *x*(*t*) as the concentration of a Mold and *u*(*t*) an Fungicide known to kill it. The differential equation representing the mold is given by

$$x'(t) = r\big(M - x(t)\big) - u(t)x(t), x(0) = x_0,$$

Where $x_0 > 0$ is the given initial population size. Note the term *u*(*t*)*x*(*t*) pulls down the rate of growth of the Mold. The effects of both the Mold and Fungicide are negative for individuals around them, so we wish to minimize both. Further, while a small amount of either is acceptable, we wish to penalize for amounts too large. Hence, our problem is as follows

$$\min_u \int_0^T Ax(t)^2 + u(t)^2 dt$$

$$\text{Subject to } x'(t) = r\big(M - x(t)\big) - u(t)x(t), x(0) = x_0.$$

The coefficient *A* is the weight parameter, balancing the relative importance of the two terms in the objective functional [5].

2.2 Numerical solution

Before writing the code we develop the optimality of this problem by first noting the Hamiltonian is

$$H = Ax^2 + u^2 + \lambda r(M - x) - \lambda xu$$

Using the optimality condition

$$0 = \frac{\partial H}{\partial u} = 2u - \lambda x \; at \; u^* \Rightarrow u^* = \frac{\lambda x}{2}$$

The adjoint equation is

$$\lambda'(t) = -\frac{\partial H}{\partial x} = -2Ax + \lambda r + \lambda u$$

$$= -2Ax + \lambda r + 0.5\lambda^2 x$$

$$= \lambda r + (0.5\lambda^2 - 2A)x$$

$$x'(t) = Mr - x(r + u) \quad , \qquad x(0) = x_0$$

$$\lambda'^{(t)} = \lambda r + (0.5\lambda^2 - 2A)x \quad , \qquad \lambda(T) = 0$$

Using these two differential equations and the representation of u^*, we generate the numerical code as described above, written in MATLAB [8].

Using the Runge- kutta sweep method [5] solving $x^{\rightarrow}$ forward in time

```
For i = 1:N
k1 = M*r - x(i)*(r + u(i));
k2 = M*r-(x(i) + h2*k1)*(r + 0.5*(u(i) + u(i+1)));
k3 = M*r-(x(i) + h2*k2)*(r + 0.5*(u(i) + u(i+1)));
k4 = M*r - (x(i) + h*k3)*(r + u(i+1));
x(i+1) = x(i) + (h/6)*(k1 + 2*k2 + 2*k3 + k4);
```

Using the Runge- kutta sweep method solving $u^{\rightarrow}$ backward in time

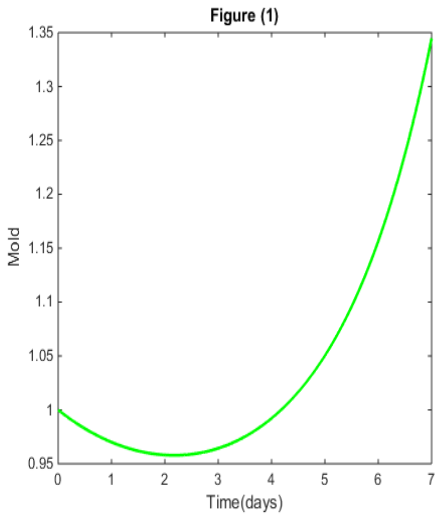
```
for i = 1:N
j = N + 2 - i;
k1 = lambda (j)*r + [0.5*(lambda (j)) ^2-2A] * x(j);
k2= (lambda (j)-h2*k1)*r + [0.5*(lambda (j) - h2*k1)^2-2A]*0.5*(x(j)+x(j-1)) ;
k3=(lambda(j)-h2*k2)*r + [0.5*(lambda(j)- h2*k2)^2-2A] * 0.5*(x(j)+x(j-1)) ;
k4 = (lambda (j)-h*k3)*r +[ 0.5*(lambda (j) - h*k3)^2-2A] * x(j-1) ;
lambda (j-1) = lambda(j) - ...
                    (h/6)*(k1 + 2*k2 + 2*k3 + k4);
```

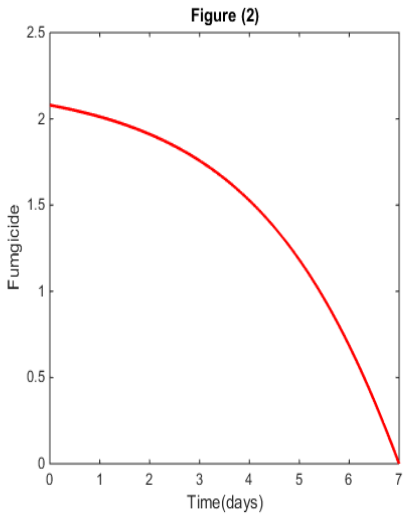
2.3 Numerical results

Here we consider a general mold and fungicide model and all the parameter values are chosen hypothetically.

Enter the values

r =0.2, M =10, A = 1, x_0 =1.

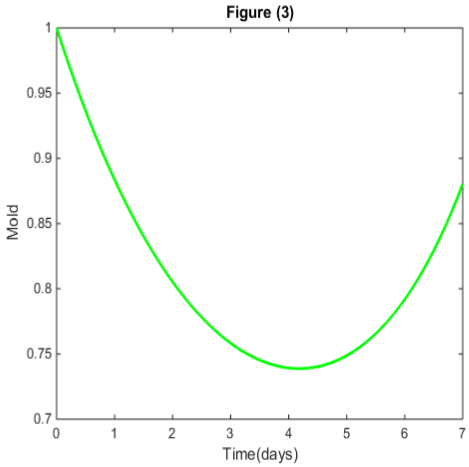


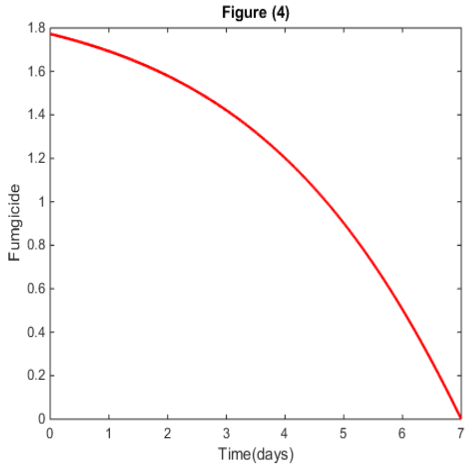


From figure (1) and figure (2) we find mold state in first and second day decrease but from third day to last day increases

Enter the values

$$r = 0.2, \quad M = 5, \quad A = 5, \quad x_0 = 1.$$

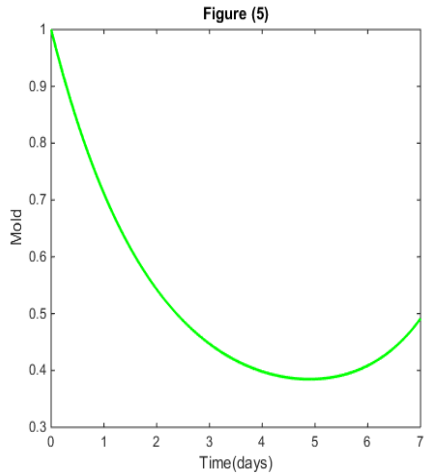


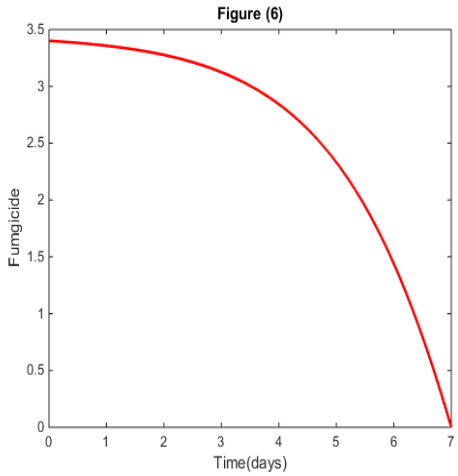


From figure (3) and figure (4) we find mold state in decrease from first day to fourth day but from third day to last day increasing mold growth

Enter the values

$$r = 0.2, \quad M = 5, \quad A = 50, \quad x_0 = 1.$$





From figure (5) and figure (6) we find mold state in decreased from first day to fifth day but from sixth day to last day increasing mold growth. We repeat this case until we get rid of the mold completely.

3-Conclusions

In this paper, the mold is sprayed every week with the fungicide until we get rid of it completely and until the houses are purified of this mold. We clean and dry the buildings as quickly as possible, as it are preferable to clean every 24 to 48 hours and use fans to help in the drying process.

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