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Adomian Decomposition Method for Application of Viscose Flows in Cylindrical Polar Coordinates

Amal F Mohamed[a] and Osman M El Mekki[b]

[a] Department of Mathematics, Shendi University, Shendi, Sudan.

[b] School of Mathematics, University of Khartoum, Khartoum, Sudan - E-mail: amaalelzbeer86@gmail.com

Abstract

In this paper we study the Adomian decomposition method for solving viscose flows in cylindrical polar coordinates. In the Cartesian coordinate system, the decomposition method was applied to the Navier-Stokes equation by Adomian. the introduced analysis is an application of the decomposition method to the Navier-Stokes equation. Also the Adomian's special polynomials was obtained through the consequent integrations.

Keywords: Decomposition method, special polynomials, Adomian's polynomials, viscose , Navier-Stokes equations, cylindrical polar coordinates.

1. Introduction

In the fluid mechanics field, the Navier-Stokes equations are considered the basic dynamical equation. These are nonlinear partial differential equations that govern the flow field of air in the ramjet, around aircraft, blood circulation in the cardiovascular system in the human body, and many other disciplines.

Solving this equation analytically is impossible because the Navier-Stokes equation is nonlinear. Some simplifications are needed to solve these equations, such as linearization or assumptions of 'weak' nonlinearity, small fluctuations, etc. Or we may need to use many traditional numerical techniques which led to huge computations.

Finding continuous analytic solutions is our objective, without huge outprints and restrictive assumptions that change the physical problem into a mathematically tractable problem. we can obtain the analytical solution to the reduced problem, but this solution will not agree with the original problem. Adomian recently introduced a powerful developed method called the Decomposition Methodology [1-3], which can provide analytic approximations to a wide class of nonlinear ordinary differential equations, systems of differential equations, partial differential equations, and systems of partial differential equations, and this method demands to be parallel to any modern supercomputer.

The decomposition method to the Navier-Stokes equations in the Cartesian coordinate system has been applied by Adomian [3]. The current analysis offers the application of the decomposition method to the Navier-Stokes equations in cylindrical coordinates utilizing which the steady two-dimensional irrotational fluid flow problems in tubes of nonuniform circular cross-sections can be studied.

In this work we used the Adomian decomposition method which applied to the Navier-Stokes equation, to obtain the Adomian's special polynomials through the consequent integrations.

2. Analysis

The Navier-Stokes equations in cylindrical coordinates which are the same as the equations of motion that govern the flow field in the tube, they are given by

$$u\frac{\partial u}{\partial z}+\upsilon\frac{\partial u}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}\right) \qquad (1)$$

Consider steady, two-dimensional motion of a viscous fluid in a tube. The equations of motion which govern the flow field in the tube are the Navier-Stokes equations

$$u\frac{\partial \upsilon}{\partial z}+\upsilon\frac{\partial \upsilon}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 \upsilon}{\partial r^2}+\frac{1}{r}\frac{\partial \upsilon}{\partial r}+\frac{\partial^2 \upsilon}{\partial z^2}-\frac{\upsilon^2}{r^2}\right) \qquad (2)$$

assuming that the rotational fluid motion is neglected. The continuity equation is

$$\frac{1}{r}\frac{\partial}{\partial r}(r\upsilon)+\frac{\partial u}{\partial z}=0 \qquad (3)$$

Here (u,υ) are the components of fluid velocity in the axial coordinate x and radial coordinate r, respectively, p is the fluid pressure, ρ is the fluid density and p is the kinematic viscosity of fluid. If we introduce the stream function ψ defined by

$$u=-\frac{1}{r}\frac{\partial \psi}{\partial r} \quad , \quad \upsilon=-\frac{1}{r}\frac{\partial \psi}{\partial z} \qquad (4)$$

the continuity equation is identically satisfied. If we eliminate p between (1) and (2), and then use the relations (4), we have the dynamical equation of motion in terms of ψ as

$$\frac{1}{r}.\frac{\partial(L\psi,\psi)}{\partial(r,z)}-\frac{2}{r^2}\frac{\partial \psi}{\partial z}L\psi=\nu L^2\psi \qquad (5)$$

where $L=\frac{\partial^2}{\partial r^2}-(1/r).\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$.

State 1:

Let $L_1 = \frac{\partial^2}{\partial r^2} - (1/r).\frac{\partial}{\partial r}$ Then the operator L becomes $L = L_1 + \frac{\partial^2}{\partial r^2}$ and equation (5) takes the form

$$L_1^2\psi = v^{-1}N\psi - \frac{\partial^4\psi}{\partial z^4} - 2\frac{\partial^2}{\partial z^2}(L_1\psi), \tag{6}$$

where $N\psi$ is the nonlinear term denoted by

$$N\psi = \frac{1}{r}.\frac{\partial(L\psi,\psi)}{\partial(r,z)} - \frac{2}{r^2}\frac{\partial\psi}{\partial z}.L\psi. \tag{7}$$

Operating with the appropriate inverse L_1^{-2} on (6) we have [1,2]

$$\psi(z,r) = \psi_0(z,r) + v^{-1}L_1^{-2}(N\psi) - 2L_1^{-2}\frac{\partial^2}{\partial z^2}(L_1\psi) - 2L_1^{-2}\left(\frac{\partial^4\psi}{\partial z^4}\right), \tag{8}$$

where ψ_0 is the solution of the homogeneous equation $L_1^{-2}\psi_0 = 0$ subject to certain boundary conditions.

In order to determine the inverse operator of L_1, we consider the equation [3]

$$L_1\psi = 0 \tag{9}$$

If we define $L_{rr} = \frac{\partial^2}{\partial r^2}$ and $L_r = \frac{\partial}{\partial r}$, then the operator L_1 becomes

$L_1 = L_{rr} - (1/r)L_r$ and equation (9) takes the form

$$L_{rr}\psi - \frac{1}{r}.L_r\psi = 0 \tag{10}$$

Solving for the linear terms $L_{rr}\psi$ and $L_r\psi$, we have

$$L_{rr}\psi = \frac{1}{r}.\, L_r\psi \tag{11}$$

$$L_r\psi = r.\, L_{rr}\psi \tag{12}$$

Operating with L_{rr}^{-1} (11) and L_r^{-1} on (12), we can write

$$\psi = \phi_1 + L_{rr}^{-1}\left(\frac{1}{r}.\, L_r\psi\right). \tag{13}$$

$$\psi = \phi_2 + L_r^{-1}\left(r.\, L_{rr}\psi\right). \tag{14}$$

where ϕ_1 and ϕ_2 are the solutions of the homogeneous equations $L_{rr}\psi = 0$ and $L_r\psi = 0$. The inverse operators L_{rr}^{-1} and L_r^{-1} are defined by

$$\begin{aligned} L_{rr}^{-1} &= \iint [\cdot]\, dr\, dr \\ L_r^{-1} &= \iint [\cdot]\, dr \end{aligned}, \tag{15}$$

Adding (13) and (14), and then dividing by 2, we have

$$\psi = \psi_0 + \frac{1}{2}\left[L_{rr}^{-1}\left(\frac{1}{r}.\, L_r\psi\right) + L_r^{-1}\left(rL_{rr}\psi\right)\right] \tag{16}$$

where $\psi_0 = (1/2)(\phi_1 + \phi_2)$. Then

$$\begin{aligned}
\psi_1 &= \frac{1}{2}\left[L_{rr}^{-1}\left(\frac{1}{r}.\,L_r\right) + L_r^{-1}\left(rL_{rr}\right)\right]\psi_0 \\
\psi_2 &= \frac{1}{2}\left[L_{rr}^{-1}\left(\frac{1}{r}.\,L_r\right) + L_r^{-1}\left(rL_{rr}\right)\right]\psi_1 \\
&\vdots \\
\psi_{n+1} &= \frac{1}{2}\left[L_{rr}^{-1}\left(\frac{1}{r}.\,L_r\right) + L_r^{-1}\left(rL_{rr}\right)\right]\psi_n
\end{aligned} \tag{17}$$

If the quantity within the bracket is denoted by S, then we can write

$$\begin{aligned}
\psi_1 &= \frac{1}{2}S\psi_0 \\
\psi_2 &= \frac{1}{2^2}S^2\psi_0 \\
&\vdots \\
\psi_{n+1} &= \frac{1}{2^{n+}}S^{n+1}\psi_0
\end{aligned} \tag{18}$$

And

$$\psi = \sum_{n=0}^{\infty}\psi_n = \sum_{n=0}^{\infty}\frac{1}{2^n}\cdot S^n\psi_0 = \sum_{n=0}^{\infty}\frac{1}{2^n}\cdot\left[L_{rr}^{-1}\left(\frac{1}{r}.\,L_r\right) + L_r^{-1}\left(rL_{rr}\psi\right)\right]_{\psi_0}^{n} \tag{19}$$

so that the inverse $L_1^{-1} = (L_{rr} - (\frac{1}{r})L_r)^{-1}$

has been identified as

$$L_1^{-1} = \sum_{n=0}^{\infty}\frac{1}{2^n}\cdot\left[L_{rr}^{-1}\left(\frac{1}{r}.\,L_r\right) + L_r^{-1}\left(rL_{rr}\psi\right)\right]^{n} \tag{20}$$

Now we can return to equation (8) and write the parametrized form of it as [2]

$$\psi(z,r)=\psi_0(z,r)+\lambda\left[v^{-1}L_1^{-2}(N\psi)-2L_1^{-2}\frac{\partial^2}{\partial z^2}(L_1\psi)-L_1^{-2}\left(\frac{\partial^4\psi}{\partial z^4}\right)\right] \tag{21}$$

Let ψ and the nonlinear term $N\psi$ be decomposed into the following forms:

$$\psi=\sum_{n=0}^{\infty}\lambda^n\psi_n \tag{22}$$

$$N\psi=\sum_{n=0}^{\infty}\lambda^n A_n \tag{23}$$

where A are special polynomials of Adomian [1,2] to be discussed later. The parameter A introduced here is not a perturbation parameter, and it is used for grouping the terms only. Then equation (21) takes the form

$$\sum_{n=0}^{\infty}\lambda^n\psi_n=\psi_0+\lambda\left[v^{-1}L_1^{-2}\sum_{n=0}^{\infty}\lambda^n A_n-2L_1^{-2}\frac{\partial^2}{\partial z^2}\left(L_1\sum_{n=0}^{\infty}\lambda^n\psi_n\right)-L_1^{-2}\left(\frac{\partial^4}{\partial z^4}\sum_{n=0}^{\infty}\lambda^n\psi_n\right)\right] \tag{24}$$

Equating the terms of like power of A, we h

$$\begin{aligned}
\psi_1&=v^{-1}L_1^{-2}A_0-2L_1^{-2}\frac{\partial^2}{\partial z^2}(L_1\psi_0)-L_1^{-2}\frac{\partial^4\psi_0}{\partial z^4}\\
\psi_2&=v^{-1}L_1^{-2}A_1-2L_1^{-2}\frac{\partial^2}{\partial z^2}(L_1\psi_1)-L_1^{-2}\frac{\partial^4\psi_1}{\partial z^4}\\
&\vdots\\
\psi_{n+1}&=v^{-1}L_1^{-2}A_n-2L_1^{-2}\frac{\partial^2}{\partial z^2}(L_1\psi_n)-L_1^{-2}\frac{\partial^4\psi_n}{\partial z^4}
\end{aligned} \tag{25}$$

The polynomials $A_0, A_1, ..., A_0$ are Adomian's polynomials (see [2]). They are defined in such a way that each An depends only on $\psi_0, \psi_1, .., \psi_n$. Thus $A_0 = A_0(\psi_0)$, $A_1 = A_1(\psi_0, \psi_1)$, $A_2 = A_2(\psi_0, \psi_1, \psi_2)$ etc.

From (6) and (23) we have

$$\sum_{n=0}^{\infty} \lambda^n A_n = \frac{1}{r}\frac{\partial(L\psi,\psi)}{\partial(r,z)} - \frac{2}{r^2}\frac{\partial\psi}{\partial z}\cdot L\psi \tag{26}$$

Substituting (22) into (26) and then equating like powers of λ from both sides of (26), we obtain

$$A_0 = \frac{1}{r}\frac{\partial(L\psi_0,\psi_0)}{\partial(r,z)} - \frac{2}{r^2}\frac{\partial\psi_0}{\partial z}\cdot L\psi_0 \tag{27}$$

$$A_1 = \left[\frac{1}{r}\frac{\partial(L\psi_1,\psi_0)}{\partial(r,z)} - \frac{2}{r^2}\frac{\partial\psi_0}{\partial z}\cdot L\psi_1\right] + \left[\frac{1}{r}\frac{\partial(L\psi_0,\psi_1)}{\partial(r,z)} - \frac{2}{r^2}\frac{\partial\psi_1}{\partial z}\cdot L\psi_0\right] \tag{28}$$

$$A_2 = \left[\frac{1}{r}\frac{\partial(L\psi_2,\psi_0)}{\partial(r,z)} - \frac{2}{r^2}\frac{\partial\psi_0}{\partial z}\cdot L\psi_2\right] + \left[\frac{1}{r}\frac{\partial(L\psi_0,\psi_2)}{\partial(r,z)} - \frac{2}{r^2}\frac{\partial\psi_2}{\partial z}\cdot L\psi_0\right] + \left[\frac{1}{r}\frac{\partial(L\psi_1,\psi_1)}{\partial(r,z)} - \frac{2}{r^2}\frac{\partial\psi_1}{\partial z}\cdot L\psi_1\right] \tag{29}$$

:If ψ_0 is once obtained, then we can obtain ψ_1 in terms of ψ_0. Similarly, ψ_2 can be obtained in terms of ψ_1 etc. So, all components of ψ are calculable and $\psi = \sum_{n=0}^{\infty} \psi_n$.

State 2:

When $L = L_{rr} - (\frac{1}{r})L_r + L_{zz}$,where $L_{zz} = \frac{\partial^2}{\partial z^2}$ equation (5) becomes

$$L^2\psi = \nu^{-1}N\psi \tag{30}$$

Operating with the inverse operator L^{-2} on (30), we get

$$\psi = \nu^{-1}L^{-2}(N\psi) \tag{31}$$

The L^{-2} involved in (31) can be obtained in the same way as L_1^{-2}, and it is given by

$$L^{-2} = \sum_{n=0}^{\infty}\frac{1}{3^n}\left[L_{rr}^{-1}(\frac{1}{r}L_r + L_{zz}) + L_r^{-1}\{r(L_{rr}+L_{zz})\} + L_{zz}^{-1}\{\frac{1}{r}(L_r + L_{rr})\right]^n \tag{32}$$

The parametrized form of equation (31) is

$$\psi = \nu^{-1}\lambda L^{-2}(N\psi) \tag{33}$$

If we substitute (12) and (23) into (33), and then equate like powers of A on both sides of (33), we obtain

$$\begin{aligned}
\psi_1 &= \nu^{-1}L^{-2}A_0\\
\psi_2 &= \nu^{-1}L^{-2}A_1\\
\psi_3 &= \nu^{-1}L^{-2}A_2\\
&\vdots\\
\psi_{n+1} &= \nu^{-1}L^{-2}A_n
\end{aligned}$$

Where $A_0, A_1, \ldots\ldots, A_n$ are Adomian's special polynomials [2] which are obtained from equations (27) – (29), etc. While using the operators L in this analysis was increased the speed of convergence, but the inverse of L and consequent integrations are more difficult [4].

$$A_0 = x$$

$$A_1 = \int\int A_0 dx = \frac{x^3}{6}$$

$$A_2 = \int\int A_1 dx = \frac{x^5}{120}$$

$$A_3 = \int\int A_2 dx = \frac{x^7}{5040}$$

$$A_4 = \int\int A_3 dx = \frac{x^9}{362880}$$

$$A_5 = \int\int A_4 dx = \frac{x^{11}}{39916800}$$

$$\vdots$$

$$A_n = \int\int A_{n-1} dx$$

The convergence of the series solution obtained by the decomposition method has been established [2,5-7] and it is also observed in [4,5] that a rapid stabilization to an acceptable accuracy is evident when numerical computation of the analytic approximation is carried out. In this section we develop an Adomian scheme to compute the solution of the full nonlinear Navier-Stokes equations through a product formula in (12) very easy to simulate since product of linear operators.

3. Conclusion

In this paper we introduce some applications of the Adomian decomposition method applied to the Navier-Stokes equation. We obtained the Adomian's special polynomials through the consequent integrations.

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